Solution to causality paradox upon total reflection in optical planar waveguide

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A dispute about the existence of an additional time associated with the Goos-Hänchen shift has recently arisen. By analyzing light propagation in an optical planar waveguide with both the zigzag-ray model and the electromagnetic theory, we show in this paper that the Goos-Hänchen time really exists, and the total time delay upon total reflection is the sum of the group delay time and the Goos-Hänchen time. The causality paradox of total reflection of a TM wave upon an ideal nonabsorbing plasma mirror is also solved with the consideration of a negative Goos-Hänchen shift.

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I. INTRODUCTION

The time delay upon total reflection has attracted considerable attention recently. A calculation [1] has predicted that the frustrated Gires-Tournois interferometer exhibits a negative delay time for total reflection, which seems to contradict causality by considering its 100% reflectivity. To solve this causality paradox, Resch *et al.* [2] showed that the Goos-Hänchen shift contributes an extra positive time, the Goos-Hänchen time, which is always large enough to make the total time delay of the frustrated Gires-Tournois interferometer positive. But recently, another example [3] is presented; that is, the total reflection of a plane *p* (or TM) wave from vacuum upon an ideal nonabsorbing plasma mirror, in which, if the Goos-Hänchen time is included, the total time delay can become negative. Based on the example mentioned above, it is indicated [3] that the existence of the Goos-Hänchen time is doubtful and the causality paradox upon total reflection remains open. And now, the situation we are faced with is that the total reflection upon the frustrated Gires-Tournois interferometer contradicts causality if the Goos-Hänchen time is not included, and the total reflection upon an ideal nonabsorbing plasma mirror violates causality if the Goos-Hänchen time is included.

In this paper, we demonstrate that the Goos-Hänchen time upon total reflection really exists. We calculate the phase of a guided mode experienced in one period of ray propagation in an optical planar waveguide with both the zigzag-ray model and the electromagnetic theory. It is shown that the zigzag-ray model coincides with the electromagnetic theory only if the phase accumulated during the Goos-Hänchen shift [2], which is the physical origin of the Goos-Hänchen time, is taken into account. Analysis of the time delay of a guided mode shows that the total time delay upon total reflection is the sum of the group delay time and the Goos-Hänchen time. Thus, causality is preserved in the frustrated Gires-Tournois interferometer. For the case of total reflection of a plane TM wave upon an ideal nonabsorbing plasma mirror, we point out that the location where total reflection occurs is not at the interface between two relevant media, but in front of it. By considering this special effect, the time delay is always positive. As a result, we suggest no problem with the relativistic causality.

II. GROUP DELAY TIME AND GOOS-HÄNCHEN TIME

In this section, we introduce the group delay time and the Goos-Hänchen time defined in Ref. [3] at first. Figure 1 shows a beam undergoing reflection from an interface at an angle of incidence beyond critical. In this diagram, Δx is the Goos-Hänchen shift and $k_x\Delta x$ is the phase accumulated during that shift, where k_x is the x component of the wave vector. The reflection phase shift φ_R [2] can be calculated by employing the Fresnel formula. We believe that this phase shift is not the total phase shift, as the Fresnel formula only deals with the components of wave vectors normal to the interface. The total phase shift φ_{tot} [2,4] upon total reflection should be written as

$$\varphi_{\text{tot}} = \varphi_R + k_x\Delta x = \varphi_R + \frac{n_1\omega}{c}\sin\theta\Delta x \quad (1)$$

where ω is angular frequency of the light, θ is angle of incidence, n_1 is index of refraction of the first medium, and c is the speed of light in vacuum. By using the stationary phase theory, the total time delay [2] is

$$\tau = \frac{\partial\varphi_R}{\partial\omega} + \frac{n_1}{c}\sin\theta\Delta x. \quad (2)$$

The time delay in Eq. (2) consists of two components. The first term, which is caused by dispersion of the reflection phase shift φ_R, is defined as the group delay time τ_g [3]. The second term is an additional contribution caused by the phase $k_x\Delta x$ and is defined as the Goos-Hänchen time. The Goos-Hänchen shift and the Goos-Hänchen time [2] can be written as

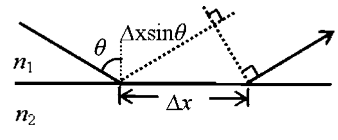

FIG. 1. Beam undergoing reflection from an interface at an angle of incidence beyond critical.

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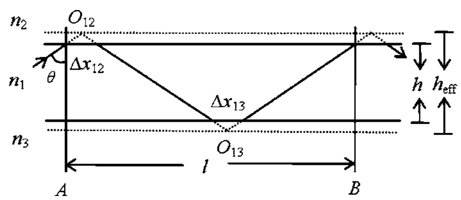

FIG. 2. Sketch of the zigzag-ray model of a planar waveguide.

$$\Delta x = -\frac{c}{n_1 \omega \cos\theta}\frac{\partial \varphi_R}{\partial \theta}, \tag{3}$$

$$\tau_{\mathrm{GH}} = \frac{n_1}{c}\sin\theta \Delta x = -\frac{\tan\theta}{\omega}\frac{\partial \varphi_R}{\partial \theta}. \tag{4}$$

If the second medium with a refractive index of n_2 is semi-infinite, the reflection phase shift of total reflection is simply expressed as $\varphi_R = \varphi_{12}$, and φ_{12} is obtained from Fresnel formula as

$$\tan\left(\frac{-\varphi_{12}}{2}\right) = \begin{cases} \dfrac{(n_1^2 \sin^2\theta - n_2^2)^{1/2}}{n_1 \cos\theta}, & \text{(TE waves)} \\ \dfrac{n_1^2}{n_2^2}\dfrac{(n_1^2 \sin^2\theta - n_2^2)^{1/2}}{n_1\cos\theta}, & \text{(TM waves)} \end{cases}. \tag{5}$$

The Goos-Hänchen shift is then obtained from Eqs. (3) and (5) as

$$\Delta x_{12} = \frac{2q_{12}\tan\theta}{k_0(n_1^2 \sin^2\theta - n_2^2)^{1/2}}, \tag{6}$$

where k_0 is the wave number of light in vacuum and q_{12} is expressed as

$$q_{12} = 1 \tag{7}$$

for TE waves and

$$q_{12} = \frac{n_1^2 n_2^2 (n_1^2 - n_2^2)}{n_2^4 n_1^2 \cos^2\theta + n_1^4 (n_1^2 \sin^2\theta - n_2^2)} \tag{8}$$

for TM waves.

III. PHASE SHIFT UPON TOTAL REFLECTION

From Sec. II, we can see that the physical origin of the Goos-Hänchen time is that the total phase shift φ_{tot} includes the phase $k_x \Delta x$ arising from the Goos-Hänchen shift. In order to confirm the existence of Goos-Hänchen time upon total reflection, we start by confirming that the total phase shift in Eq. (1) is correct.

Figure 2 shows an optical planar waveguide where a guiding layer of high refractive index is sandwiched between two semi-infinite cladding layers of low refractive indices. Guiding is achieved by total reflection of the optical rays upon two cladding layers. The dispersion relation of a guided mode for both polarizations is given by

$$2\kappa h + \varphi_{12} + \varphi_{13} = 2m\pi \tag{9}$$

with

$$\kappa = (k_0^2 n_1^2 - \beta^2)^{1/2} = k_0 n_1 \cos\theta, \tag{10}$$

$$\tan\left(\frac{-\varphi_{1j}}{2}\right) = \begin{cases} \left(\dfrac{N_m^2 - n_j^2}{n_1^2 - N_m^2}\right)^{1/2} & \text{(TE modes)} \\ \dfrac{n_1^2}{n_j^2}\left(\dfrac{N_m^2 - n_j^2}{n_1^2 - N_m^2}\right)^{1/2} & \text{(TM modes)} \end{cases} \tag{11}$$

$$\beta = k_0 N_m = k_0 n_1 \sin\theta, \tag{12}$$

where m is the mode order, N_m is the effective index of the guided mode, h is the thickness of the guiding layer, β is propagation constant, κ is the component of wave vector normal to the propagation direction in guiding layer, θ is the angle of incidence, and $j=2,3$ represents two cladding layers, respectively.

We consider the propagation of a guided mode with a propagation constant of β. According to the electromagnetic theory of planar waveguide, the field distribution of the guided mode can be expressed as

$$E(x,y,t) = E(y)\exp[i(\beta x - \omega t)], \tag{13}$$

where $E(y)$ is the amplitude of the guided mode. Figure 2 illustrates the propagation of the guided mode by employing the zigzag-ray model. The distance between A and B, which corresponds to one period of propagation of the zigzag rays in guiding layer, is written as

$$l = 2h\tan\theta + \Delta x_{12} + \Delta x_{13}, \tag{14}$$

where Δx_{12} and Δx_{13} are the Goos-Hänchen shifts at two boundaries. By considering Eq. (13), the phase shift of the guided mode in this distance is

$$\phi_{\mathrm{mode}} = \beta l. \tag{15}$$

In zigzag-ray model, the total phase shift of the rays in this range is obtained from Eq. (1) as

$$\phi_{\mathrm{ray}} = 2k_0 n_1 \frac{h}{\cos\theta} + \varphi_{12} + \beta\Delta x_{12} + \varphi_{13} + \beta \Delta x_{13}, \tag{16}$$

which can be expanded as

$$\phi_{\mathrm{ray}} = 2k_0 n_1 h\cos\theta + 2k_0 n_1 h \sin\theta \tan\theta + \varphi_{12} + \beta\Delta x_{12} + \varphi_{13} + \beta\Delta x_{13}. \tag{17}$$

Substituting Eqs. (10), (12), and (14) into Eq. (17), we obtain

$$\phi_{\mathrm{ray}} = (2\kappa h + \varphi_{12} + \varphi_{13}) + \beta l. \tag{18}$$

The first term in the right-hand side of Eq. (18) equals $2m\pi$ for a guided mode, which is exactly the dispersion relation of Eq. (9), and the second term is the same as the phase shift in Eq. (15) obtained from the electromagnetic theory. Therefore, the zigzag-ray model of the waveguide coincides with the electromagnetic theory only at the condition that the phases arising from the Goos-Hänchen shifts are included. And then, the expression of total phase shift in Eq. (1) is correct.

IV. THE EXISTENCE OF THE GOOS-HÄNCHEN TIME UPON TOTAL REFLECTION

In this section, we confirm the existence of the Goos-Hänchen time by employing the same planar waveguide as described in Fig. 2. For simplicity, the guiding layer with refractive index of n_1 and two cladding layers with refractive indices of n_2 and n_3 are assumed without dispersion. In this case, the group delay times of total reflection at two boundaries of the guiding layer are

$$\tau_g = \frac{\partial \varphi_{1j}}{\partial \omega} = 0, \quad j = 2,3, \tag{19}$$

and the Goos-Hänchen times alone are the total time delays upon total reflection. We also consider a guided mode with an effective index of $N_m = n_1 \sin\theta$. The Goos-Hänchen shifts at two boundaries are rewritten from Eq. (6) as

$$\Delta x_{1j} = \frac{2q_{1j}\tan\theta}{k_0(N_m^2 - n_j^2)^{1/2}}, \quad j = 2,3 \tag{20}$$

with

$$q_{1j} = 1 \tag{21}$$

for TE modes and

$$q_{1j} = \frac{n_1^2 n_j^2 (n_1^2 - n_j^2)}{n_j^4(n_1^2 - N_m^2) + n_1^4(N_m^2 - n_j^2)} \tag{22}$$

for TM modes. Expression (20) is also the general form of the Goos-Hänchen shift for total reflection upon a semi-infinite medium. For both polarizations, substituting Eq. (20) into Eq. (14), we obtain

$$l = 2h_{\mathrm{eff}}\tan\theta, \tag{23}$$

where h_{eff} is the effective thickness of the guided mode [5,6] and has the form of

$$h_{\mathrm{eff}} = h + \frac{q_{12}}{k_0(N_m^2 - n_2^2)^{1/2}} + \frac{q_{13}}{k_0(N_m^2 - n_3^3)^{1/2}}. \tag{24}$$

Although Eq. (23) has the same form for both TE and TM modes, the effective thicknesses of the TE and the TM modes are different. If we extend the incident and reflected rays into two cladding layers, they cross at points O_{12} and O_{13} in Fig. 2, respectively. And then, we can see from Eq. (23) and Fig. 2 that the rays behave as if they were propagating in a guide of effective thickness [5,6].

The propagation time of the guided mode in length l can be calculated by two methods: (i) the zigzag-ray model takes account of Goos-Hänchen times that occur at the guiding layer boundaries and (ii) the electromagnetic theory determines the propagation time by dividing the length l with the group velocity. Since the group velocity of the guided mode is derived from the rigid electromagnetic theory, if the times calculated from two methods are equal, the existence of Goos-Hänchen time is confirmed.

By employing Eqs. (4) and (20), the Goos-Hänchen times that occur at two boundaries are

$$\tau_{\mathrm{GH}}^{(1,j)} = \frac{2n_1 q_{1j}\sin\theta\tan\theta}{ck_0(N_m^2 - n_j^2)^{1/2}} = \frac{2N_m^2}{c(n_1^2 - N_m^2)^{1/2}}\frac{q_{1j}}{k_0(N_m^2 - n_j^2)^{1/2}}. \tag{25}$$

It is should be noted that the same expression as Eq. (25) has been obtained in Ref. [5] by using $\partial\varphi_{1j}/\partial\omega$ in a waveguide. But in derivations of Ref. [5], the authors treat the propagation constant as an independent variable and think that the partial derivative of the propagation constant with respect to the angular frequency is zero. Those derivations are correct only for guided modes in a waveguide and cannot be applied to the general case of total reflection upon a semi-infinite medium. Thus, the physical meaning of Eq. (25) is not the group delay time defined in Ref. [3], but the Goos-Hänchen time defined in Eq. (4).

By considering the zigzag-ray model as shown in Fig. 2, the optical path of the rays in guiding layer in the range between A and B is $2n_1 h/\cos\theta$, and the propagation time is

$$\tau_{\mathrm{ray}} = \frac{2hn_1}{c\cos\theta} = \frac{2hn_1^2}{c(n_1^2 - N_m^2)^{1/2}}. \tag{26}$$

As a result, the time delay of the rays in length l calculated from method (i) is given by the sum of τ_{ray} and two Goos-Hänchen times; that is

$$\tau_{\mathrm{tot}} = \tau_{\mathrm{ray}} + \tau_{\mathrm{GH}}^{(1,2)} + \tau_{\mathrm{GH}}^{(1,3)} = \frac{2N_m^2 h_{\mathrm{eff}}}{c(n_1^2 - N_m^2)^{1/2}} + \frac{2h(n_1^2 - N_m^2)^{1/2}}{c}. \tag{27}$$

Next, we use method (ii) to calculate the propagation time of the guided mode in the length l. The group velocity of the guided mode is given by

$$\frac{1}{v_g} = \frac{N_m}{c} + k_0\frac{\partial N_m}{\partial\omega} = \frac{N_m}{c} + \frac{(n_1^2 - N_m^2)h}{cN_m h_{\mathrm{eff}}}, \tag{28}$$

where v_g is the group velocity of the guided mode and $\partial N_m/\partial\omega$ is calculated by performing the partial derivative of dispersion relation with respect to the angle frequency. The group velocity can also be obtained by employing the relation $v_g = P/W$, where P is the power flow and W is the stored energy of the waveguide [5]. For both TE and TM polarizations, the time delay of the guided mode in the length l is

$$\tau_{\mathrm{mode}} = \frac{l}{v_g} = \frac{2N_m^2 h_{\mathrm{eff}}}{c(n_1^2 - N_m^2)^{1/2}} + \frac{2h(n_1^2 - N_m^2)^{1/2}}{c}. \tag{29}$$

From Eqs. (27) and (29), it is shown that, only if the Goos-Hänchen times are taken into account, does the zigzag-ray model of waveguide coincide with the electromagnetic theory. This example indicates that the Goos-Hänchen time upon total reflection really exists. If $\partial\varphi_R/\partial\omega$ is not zero, the total time delay upon total reflection is the sum of the group delay time and the Goos-Hänchen time. Thus, causality is preserved in the frustrated Gires-Tournois interferometer [2].

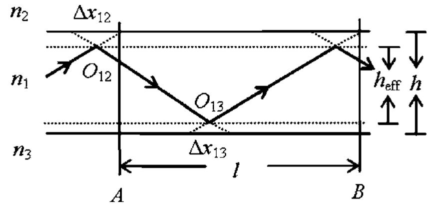

FIG. 3. Zigzag propagation of the rays in planar waveguide with both cladding layers owning negative permittivities.

V. TOTAL REFLECTION OF A TM WAVE UPON A MEDIUM WITH NEGATIVE PERMITTIVITY

The total reflection of a TM wave upon a medium with negative permittivity,

$$\varepsilon_2 = n_2^2 < 0 \tag{30}$$

deserves special consideration. In this case, we can see from Eqs. (6) and (8) that the Goos-Hänchen shift Δx_{12} is negative. The Goos-Hänchen time τ_{GH} is also negative, which can result in a negative total time delay and seems to violate relativistic causality, for example, the total reflection of a plane TM wave from vacuum upon an ideal nonabsorbing plasma mirror [3].

In fact, this is not the case. Let us consider a planar waveguide where the guiding layer has a positive refractive index n_1 and two cladding layers have negative permittivities of $\varepsilon_2 = n_2^2 < 0$ and $\varepsilon_3 = n_3^2 < 0$. For simplicity, we also assume that n_1, n_2, and n_3 are without dispersion. As shown in Fig. 3, the effective thickness of a TM mode has the relation of

$$h_{\text{eff}} = h + \frac{q_{12}}{k_0(N_m^2 - n_2^2)^{1/2}} + \frac{q_{13}}{k_0(N_m^2 - n_3^2)^{1/2}} < h. \tag{31}$$

With the same procedures in Sec. IV, the same expressions as Eqs. (27) and (29) can also be obtained. Therefore, the negative Goos-Hänchen times at two boundaries coincide as well with the electromagnetic theory of the waveguide.

Next, we will show that the negative Goos-Hänchen time does not contradict causality. Because of the negative Goos-Hänchen shifts at two boundaries of the guiding layer, $h_{\text{eff}} < h$ holds true, and the two crossing points O_{12} and O_{13} in Fig. 3 are not located in the cladding layers, but in the guiding layer. According to electromagnetic theory, because of negative permittivity, the direction of the time-averaged Poynting vector and its flux lines [7] in the two cladding layers of the slab waveguide in Fig. 3 is opposite to the propagation direction of the guided mode, resulting in the total power flow of the guided mode concentrating in the range of effective thickness [5,6] that is less than the thickness of the guiding layer. In ray model, this effect can only be explained as that total reflections occur at points O_{12} and O_{13}. This conclusion is also supported by Ref. [7] which calculates time-averaged Poynting vectors and its flux lines in the two media for a Gaussian beam in the TM state in the case of a negative Goos-Hänchen shift. It is shown that the incoming flux lines do not pass through the interface between two media at all, but are all located in the first medium, and the intersection of the incident beam axis and the reflected beam axis is exactly the crossing point O_{12} or O_{13} shown in Fig. 3. From this point of view, the total reflection of a TM wave upon a medium with negative permittivity is shown in Fig. 4. We can see from Fig. 4 that the negative Goos-Hänchen shift and time are reasonable because the point P_1 is selected as reference point, whereas the total reflection occurs at the point O. Equation (4) indicates that the Goos-Hänchen time is exactly the amount of time it would take a light wave front to move a distance $\Delta x_{12} \sin\theta$. So the Goos-Hänchen time in Fig. 4 is $-n_1 P_1 P_3 / c$. The times of light propagation from O to P_1 and from P_2 to O are $n_1 OP_1/c$ and $n_1 P_2 O/c$. As a result, if we calculate the time delay at the point O where the total reflection occurs, the time delay is

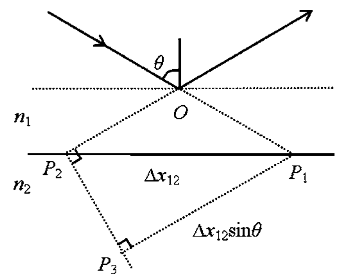

FIG. 4. Total reflection with a negative Goos-Hänchen shift.

$$\begin{aligned}\tau'_{GH} &= \frac{n_1(OP_1 + P_2O - P_1P_3)}{c} \\ &= \frac{n_1 \Delta x_{12}(\sin\theta - 1/\sin\theta)}{c} \\ &= \frac{-2n_1 q_{12}\cos\theta}{\omega(n_1^2\sin^2\theta - n_2^2)^{1/2}},\end{aligned} \tag{32}$$

which is always positive by considering $OP_1 + P_2O > P_1P_3$ or the negative Δx_{12}.

In order to testify Eq. (32), the time delay of a TM mode during one period of ray propagation in the waveguide illustrated in Fig. 3 is recalculated by considering that the optical rays are reflected at the points O_{12} and O_{13} with the time delays at two points given by Eq. (32). The optical path of the rays in length l is then $2n_1 h_{\text{eff}}/\cos\theta$. By considering Eqs. (32) and (24), and $N_m = n_1 \sin\theta$, the time delay in length l is written as

$$\begin{aligned}\tau_{\text{tot}} &= \frac{2n_1 h_{\text{eff}}}{c\cos\theta} - \frac{2n_1 q_{12}\cos\theta}{\omega(N_m^2 - n_2^2)^{1/2}} - \frac{2n_1 q_{13}\cos\theta}{\omega(N_m^2 - n_3^2)^{1/2}} \\ &= \frac{2n_1^2 h_{\text{eff}}}{c(n_1^2 - N_m^2)^{1/2}} - \frac{2(n_1^2 - N_m^2)^{1/2} q_{12}}{ck_0(N_m^2 - n_2^2)^{1/2}} - \frac{2(n_1^2 - N_m^2)^{1/2} q_{13}}{ck_0(N_m^2 - n_3^2)^{1/2}} \\ &= \frac{2N_m^2 h_{\text{eff}}}{c(n_1^2 - N_m^2)^{1/2}} + \frac{2h(n_1^2 - N_m^2)^{1/2}}{c}.\end{aligned} \tag{33}$$

The equivalence of Eqs. (33) and (29) shows that, for total reflection with a negative Goos-Hänchen shift, the Goos-Hänchen time should be expressed as Eq. (33) and the location where total reflection occurs lies in the crossing point of the incident ray and the reflected ray. Therefore, by considering this effect, the causality paradox does not exist.

In this paper, we deal with the total reflection of continuous and monochromatic light. Therefore, in the case of total reflection with a negative Goos-Hänchen shift, the closed-loop flux lines [7] around the interface have already existed. Since the amplitude of the incident light is invariant in time, there is no energy exchange between the closed-loop flux lines and the flux lines of the incident and reflected lights. But the electromagnetic fields around the interface, which result in the closed-loop flux lines, and the fields of the incident and reflected lights will interact with each other, resulting in that total reflection occurs before the light reaches the interface. In waveguide, this explains why the effective thickness of the guided mode is less than the thickness of the guiding layer. Although the light is reflected before it reaches the interface, we believe that the light knows the interface is there by considering that the energy of the closed-loop flux lines around the interface also comes from the incident light and all fields (the fields around the interface and the fields of incident and reflected lights) must comply with the Maxwell's equation. In the case of light pulses incidence, energy transfer will occur between the energy around the interface and the energies of incident and reflected pulses, and the result will be different.

VI. CONCLUSION

By employing an optical planar waveguide with the cladding layers of positive or negative permittivities, we have confirmed the existence of Goos-Hänchen time upon total reflection. The time delay of total reflection is the sum of the Goos-Hänchen time and the group delay time. The Goos-Hänchen time is always positive, suggesting that there is no problem with the relativistic causality in the frustrated Gires-Tournois interferometer and the total reflection of a TM wave upon an ideal nonabsorbing plasma mirror.

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